computability in the light of the Master Argument

PhD Kolloquium WS 2012

vera bühlmann, December 4th 2012

from *linear algebra* to *algebraic invariances*

properties that remain stable within a system under transformations

a *linear equation* is an equation in which each term is either a constant or the product of a constant and a single variable (not raised to any power).

Linear Algebra studies systems of such equations.

In linear algebra, the **determinant** is a *value associated with a square matrix*. It can be computed from the entries of the matrix.

Before the study of invariances, the study of *linear equations* was usually subsumed under that of *determinants*

No consideration was given to *systems in which the number of equations differed from the number of unknowns*. This changed with the interest in invariances.

Quantics

transformations carried out on the variables or coefficients („constants“) within *algebraic forms*

> not the *solution* of a system of equations, but the solv*ability* of equations and their systems.

key terms in linear algebra

field - rational domain within the compelx numbers satifying certain conditions

vector - establishes the operability over a field, n-dimensionality, in ideal theory vectorspaces are taken as subsets in rings.

ring - establishes unique factorization, modularity, of a field

algebra - hypercomplex number system (a ring and a vector space over a field)

what about space and geometry in linear algebra ?

they are dealing with *invariances* - properties that remain stable under transformations operating on the variables or coefficients within *algebraic forms*.

> what is the nature of these invariances ?
Properties of *what* are we dealing with? This is the main disconcertion associated with invariances from a non-technical point of view.

Space has always given us the opportunity to identify properties of „things“ - now, what does that mean for our notions of object and subject (logics) when we speak of „properties of spaces“ in topology?

algebra

handle this with special care - the distinctions are assorted according to my own reasoning! no standard list of distinctions!

quantics linear algebra (forms of homogenous transformations) - invariance and covariance, vector spaces and matrices. no systematization, a cookbook.

abstract algebra Interest in the solvability of structures in abstraction from any specific „ground“ - through the conception of the ground in ideality, rings, fields, modules, groups, etc.

symbolic algebra Interested in providing the conditions for solvability. The ground is symbolic and engendered (not assuming a *given* „nature“ of numbers)

symbolico-physical treats valences in physics and chemistry, according to principles of *saturation* (from the Latin word *saturare*, meaning *to fill*) - a notion of fullfillment based on reaching a maximum capacity.

> Doping technologies.

Aristotle‘s dynamics of privation !

E.T. Bell, in *The Development of Mathematics*

chapter on Invariances

1 General Features of Invariance
2 algebraic invariance
3 the synthesis by transformation groups
4 codification of geometry by invariance
5 intrinsic spatial invariance

"Invariance is changelessness in the midst of change, permanence in a world of flux, the persistence of configurations that remain the same despite the swirl and stress of countless hosts of curious transformations."
(E.T. BELL p. 420)

paragraph 1 General Features of Invariance

along which *path*?

Quantics - the study of *algebraic form*

⇩

Abstract and Symbolic Algebra - an undiscovered *continent*
George Boole

⇩

Relativism - the *hypotheses at the foundation of geometry*
Bernhard Riemann

⇩

Systematization - through *seeking a unification of geometries*
Felix Klein

⇩

Physics - *general theory of relativity* (Conservation Laws)
Albert Einstein

encircling which *problems*?

Invariance as *new unifying principle*, abstraction from the cook book listing of calculations in the algebra of forms

the old unifying principle were the *elementary forms* in Euclidean space, and with Descarte their analytical description

Riemann Space - a *cosmic* imagination, against any *absolutist* imagination

what does that mean for the role of the imagination within thought? > *From Descartes to Riemann*

invariant theory as an *addition* to mathematical thought ?

what is that supposed to mean? categorical difference between quantities and symbols

paragraph 2 Algebraic Invariance

Determinants - evolution of the symbolic method

from the *Calculus of Quantics* to the *Algebra of Quantics*

from invariance as *a property of quantities* to invariance *as the porperty of groups.*

settling in the *undiscovered continent* - devices of calculation bring about „new Provinces“:

foundations of mathematics
algebraic geometry (encoding of space and space-time)
chemico-algebraic theory (theory of chemical valence e.g. benzene)
-> quantics was eventually absorbed into *quantum theory*

Hermann Weyl:
the ‚quantities‘ are vectors appertaining to specific representations of a group – calculation of *group characters*, (algebra of all matrices commuting with every matrix of the given algebra)

Problems of Quantics – is there *a fundamental system*, and *a finite set of independent irreducibles*?

for quantics YES, but for quantum theory NO!

cf the Hilbert - Brouwer/Weyl controversy. *Intuitionism.*

settling in the undiscovered continent (Algebra) was interpreted *politically* before breaking a battlefield between ideological claims for supremacy.

Bell speaks of an „army of algebraists and projective geometers“ who storm into „the fertile territory“ of abstract symbolic algebra. Those who applied the *symbolic method* were adventurers stigmatized as „illegitimate Kings“ striving for „profit“ in the domains of their settlement: Bell writes how they were „recruting masses of young mathematicians who mistake the kingdom of quantics for the democracy of mathematics“.

why is the application of symbolic methods a *kingdom* (Monarchy) and not a *democracy* (Republic)

Quantics, Algebraic Form

Are the realms of abstraction „territory“ – if it is an undiscovered continent“ to be *conquered*?

L. *conquirere* for "to search for, procure by effort, win" from *com-* for "together, together with, in combination," and *quaerere* "to seek, acquire"

conquer -> to subject (as a verb)
to subject means „to render submissive or dependent," "person under control or dominion of another," "person or thing that may be acted upon" "subject matter of an art or science".

Are adventurers in abstraction supposed to be like seafarers in Renaissance, obliged to *sail in the name* of the Queen alias the State?
> then the State turns into a monarch of the multitude, capable of tyranny and terror!

Enlightenment – political secularization

science detached from metaphysics/theology

The Algebraists were accused of *totalitarian calulation:* as campaigning to recruit mathematicians for theory with no application (useless)

„Cayley's numerous successes, quikly followed by those of the prolific Sylvester, unleashed one of the most ruthless *campaigns of totalitarian calculation in mathematical history.* [...] Such misdirected foresight was not peculiar to the algebra of quantics in mathematics since 1850. In the accompanying theory of groups, for example, especially permutation groups, there was a similar panic. Once *the means for raising unlimited supplies of a certain crop are available,* it would seem to be an excess of caution to keep on producing it till the storehouses burst, unless, of course, the crop is to be consumed by somebody. There have been but few consumers for the calculations mentioned, and none for any but the most easily digested. Nevertheless, the campaign of calculation for the sake, apparently, of mere calculation did at least hint at undiscovered provinces in algebra, geometry, and analysis that were to retain their freshness for decades after the modern higher algebra of the 1870's had been relegated to the dustier classics.“

(E.T.Bell p. 429/30)

> fear of the potential applications that were made available

Bell‘s stance: criticizes the filling of the „storages“ with „intellectual nourishment“ that hardly anyone can digest.

non-absoluteness of the Law of the Excluded Middle

Intuitionism

Background to the Hilbert – Brouwer/Weyl controversy

„For this memorable victory over the barbaric hordes of algebraic formulas, Gordan was crowned "King of Invariants" [...] He occupied the throne exactly twenty-two years, until Hilbert, a mere stripling of twenty-eight, in 1890 snatched the crown from Gordan's ageing head and rammed it firmly down on his own.“ (E.T.Bell)

"This is not mathematics; it is theology"

Gordon on Hilbert

Gordan in 1868 proved the existence of finite fundamental systems of invariants and covariants for any binary quantic, and in 1870 did the same for systems of such forms. His method was constructive, he demonstrated a procedure which can generate all the instances.

finitist constructive method

Hilbert extended the finite theorem significantly by giving a purely formal proof without procedure to actually generate all instances. On purely logical grounds – by applying the Law of the Excluded Middle to Cantor‘s infinite sets.

finitist formalist method

contradictories for the accidental properties, Contradiction for the essential properties.

Jan Brouwer and Herman Weyl (both students of Hilbert) critizized the formalist (logical) method and instist in *infinitary methods* when dealing with infinities.

infinitary methods

Aristotle‘s privation-dynamics is a prototype for infinitary method; the same for Booles‘ Algebra of logics, and for Dedekind‘s and Noether‘s conceptual approaches.

different treatments of transformation groups

paragraph 3 The synthesis by transformation group

Structural theory – inversion of problem-solving approaches: not is there a solution to a certain problem, but which operations are sufficient and necessary, and what mathematical objects must be invented, *to provide a solution for a problem of a prescribed kind.*

elimination through composition series (Lie - reduction to linearity),
and algebraic characterization of the nature of irrationals (fields, domains, etc) directly on the systems (Weyl)

looking for integrability – The primary objectives (of algebraic structural theory) are *to discover what can be done rather than to do it*, and to *give criteria for what cannot be done.*

Distinguish between *reasonable formulatio of problems* and *not reasonable formulation of* problems (no or many solutions).

physical and chemical valences (doping) – construct manifestations of invariants for systems of (partial) equations whose conditions the abstract invariant must satisfy.

Organic and non-organic chemistry, particle physics.

logistics, geodesics, analytical mechanics – application of invariants to kinematics, Helmholz (describe Euclidean Space kinematically (not static!) by working with differential invariants. Lie secured linearization for differential equation like Newton did for infinitesimal curves.

operation research, cybernetics

paragraph 4 The codification of geometry

Codification - from Klein to Hilbert

Paradigms of langauge, information, quantization

from *kinematic space* to *quantum space* - from *equivalence* transformations between groups that are made commensurable by schemata (quantified), to transformations within groups that are *identities* (Riemann) and need to be *quantized*, not *quantified*.

search for *a unified field theory* - abstracting from Einstein‘s *gravitational field* and Maxwell‘s electromagnetic field – *attempts* developed by Weyl, leading to *quantum mechanics*.

point-set geometries – geodesics, Levi Civita parallel displacement (the sum of the angles of a triangle on the earth is more than 180°), generalization to the geometry of path, eventually: *break with linearity of connections* (Dirac‘s *Algebra of Quantum Theory*) *codification of geometry*

The Erlangen Program 1872

(*Felix Klein 1872*)

„A *geometry* is defined as the system of definitions and theorems invariant under a given group of transformations.

Two groups of transformations are to be considered in connection with any geometrical relation: a *group by means of which the relation may be defined*; a group *under which the relation is left invariant*. The more restricted the group, the more figures will be distinct relatively to it, and the more theorems will appear in the geometry.

The extreme case is *the group corresponding to the identity* [the transformation leaving everything considered invariant], the *geometry of which is too large to be of consequence*."

note the indefinite article –
a geometry, like *a* language

group - structural solution space (can be expresses as an axiomatic system (Euclid))

If at least *one postulate* of a mathematical (hypothetico-deductive) system *be suppressed*, the system developed from the modified set of postulates is *less restricted than the original*. In this sense, the modified system is *more fundamental and more general* than the original system.

all the properties of a space would be considered if an identity relation is applied instead of equivalence relations

Riemann Space

Klein/Hilbert

logic here has a transcendental role - this was the main critique of Wittgenstein in the Tractatus!

(and Felix Klein) the *definition of sets by code, working with representing groups by summation of subspaces within coordinate systems*

Klein‘s'Erlanger Programm' of 1872 for the codification of geometry as it existed at the time, essentially *reduced geometry to the study of equivalence under certain groups of transformations.* In the modernized presentation of Klein's project (e.g. in Hilbert axiomatization of mathematics), groups appear as groups of automorphisms and of *preferred coordinate systems of the base space*. The automorphisms leave invariant the characteristic relations of the particular geometry concerned. These automorphisms induce *transformations on the vector subspaces of the base space*.

this needs a *representation* of *groups as objects* – and assumes coordinate space. i.e. it cannot deal with Riemann space, only with representations of regions therein.

Weyl

the metrics in a Riemann space is qualitative - the distance function cannot be anchored „objectively“ in coordinates

the *characterization of groups* through *invariant integrals* (not summation of subspaces)

In the calculation of group characters, analytical considerations offer an alternative to finite summations of subsets, which are being replaced by *integrations over the group manifold*, the invariant element of volume of a compact group having been suitably defined.

is purely operational and works directly on Riemann spaces (non-coordinated spaces, manifolds).

„This [Klein's] point of view was the dominant one for the first half century after it was enunciated. It was a helpful guide in actual study and research. Geometers felt that it was a correct general formulation of what they were trying to do. For they were all thinking of space as a locus in which figures were moved about and compared [as was implied earlier in connection with Lie's revision of Helmholtz' kinematic geometry]. The *nature of this mobility* was what distinguished between geometries.“ (E.T.Bell)

space as a locus in which figures were moved about and compared

„nature of this mobility“ > Kinematics is the branch of classical mechanics that describes the motion of points, bodies (objects) and systems of bodies (groups of objects) without consideration of the causes of motion.

No unified field theory for the space of a geometry:

Einstein‘s field of Gravitation and Maxwell‘s field Electromagnetism

This brought to attention precisely those Riemannian geometries about which the Erlanger Programm said nothing, namely *those whose group is the identity.* In such spaces there is essentially only one figure, namely the space structure as a whole.

With the advent of *Relativity* we became conscious that space need not be looked at only as a "locus in which," but that it may *have a structure, a field-theory*, of its own.

Einstein‘s Relativity Theory applied Riemann‘s geometry > space-time.

metrics

„A *geometry* is defined as the system of definitions and theorems invariant under a given group of transformations".

space

"A space is a set of objects with a definite system of properties, called the structure of the space." (cited in Bell p. 448)

Space is not a *locus in which objects* are placed and *subjects* act (governed by whom/whatever).

> this is what is so silly about *Object-Oriented Philosophy* Movement! (Object Oriented Design is simply a technical / descriptive term)

they are dealing with quantification, not with quanization. As if philosophy were game design.

matter

With relativity theory – *matter* itself becomes an *aspect of spacetime*. The measurement of curvature varies from point to point in a manner corresponding to the amount of matter present. In the absence of matter, spacetime is flat.

Coding – *quantification* vs *quantization*

quantization is where we don‘t need to have a model of language-in-general!
life streams and SOMs (self-organizing maps) breeding clusters of dimensionalities that can be interpreted and formalized into groups.

in codification theory:

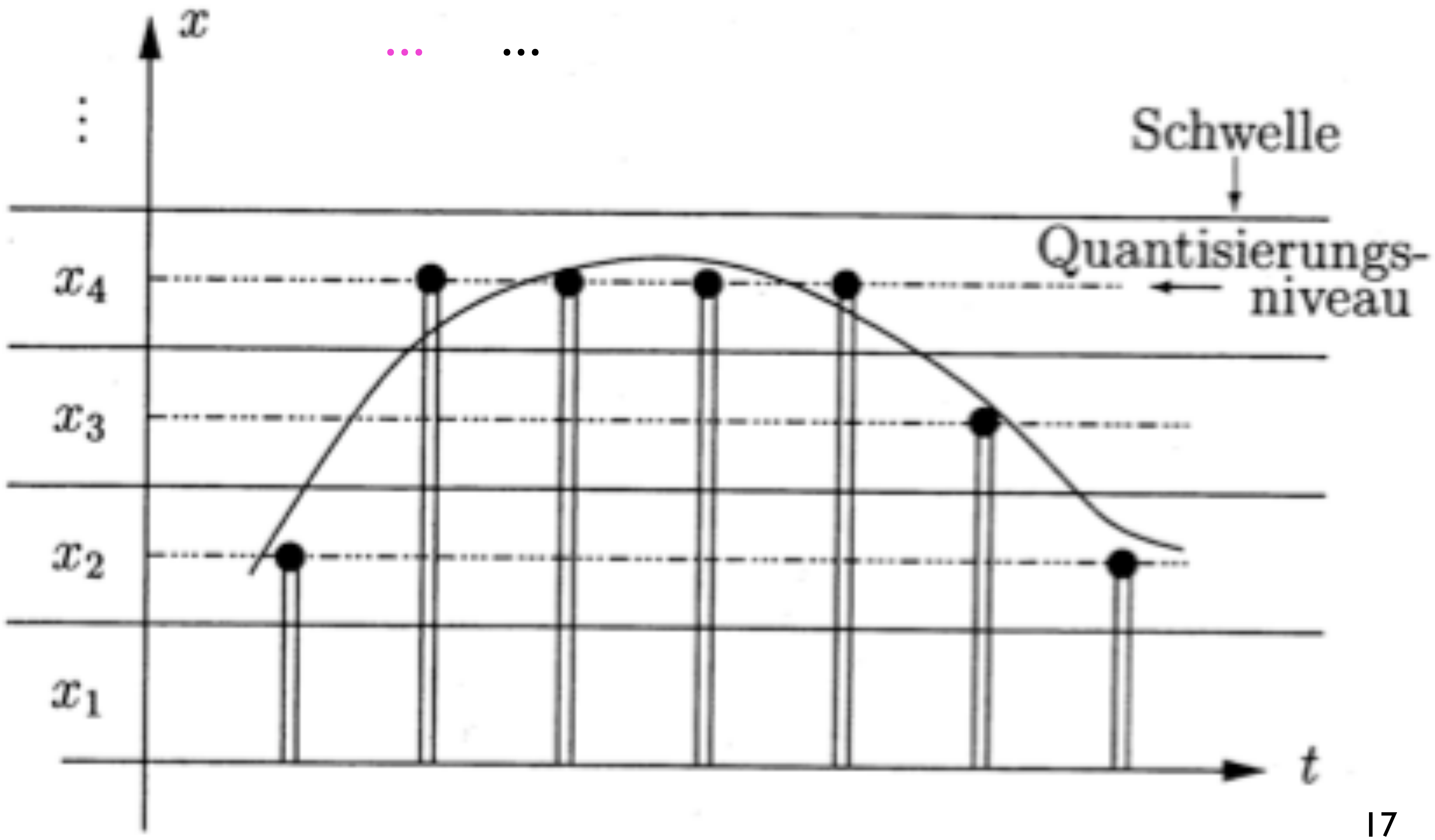
transformation groups as *alphabets*

{a, b, c .... x, y, z} alphabet in letters

{0, 1, 2,.....8, 9} decimales alphabet

{0, 1} binary alphabet

... ...



formal Language vs *Quantum* paradigm

A „word“ is *a sequence of signs* over an *alphabet of signs* as a *finite group of components closed by rules of operation* (syntax/arithmetics)

change of an alphabet

e.g. analog/digital converter

involves *double discretization* of amplitude frequencies:

1) sampling (Abtastung)

2) quantization (Quantelung)

paragraph 5 intrinsic spatial invariance

topology - can it be interpreted as „the royal road to mathematics“ ?

„the devil of abstract algebra and the angel of topology“ (Herman Weyl): one striving for articulation and differentiation, the other for unification.

⇩

properties of a space - topology works with its own categories of how to articulate space

neighborhood, region, bounary, etc. These allow to predicate the properties of a space constructed formally.

⇩

topological space - topology constructs its spaces according to the transformation groups of its objects

i.e. by the qualitative properties of space (independent of size, location, and shape): continous, homeomorphic (finite and continuous) correspondences.

⇩

knots – structural „objects“ (knots) in non-coordinated space (networks) can be analyzed within an aspectual space only (their appearances). They have relative dimensionality (topological invariances, can complexes or simplexes.

Analysis allows to enumerate and characterize all possible knots (Euler‘s bridge problem) relative to the preserved invariances.

⇩

topology illustrates properties of functions – appearances are constituted by their qualitative dimensionality

⇩

topology in analysis - inversion of Descartes analytical method

Accidential properties (dimensions in topology) for Descartes were *series*, starting from an *absolute* point. Construction had to build up from simple to complex. Now we can postulate properties as invariants of groups of complex bodies, and then construct the spaces accordingly. In complex analysis - we code with the imaginary numbers in the real numbers (Dedekind Cut). The role of imagination changes!

an Aristotelian mindset and contemporary mathematics ?

recap the dynamics of privation

Everything is subject to privation in the events that can happen to them, they gain their individuality thereby.

The space of quality is full – but never exhaustible!

He ascribed privation to events.
Can we ascribe it to subjects ?

Is this *real* or *virtual?*

Aristotle‘s *future contingents, as in the example below*

abstraction can accomodate more diversities. abstraction brings relaxation.

Aristotle‘s example of probabilistic truth value based on opinion:

A constradiction, if it concerns the accidential, can be treated operationally and can be harvested in the dynamics it unfold – in *ethics*.

theory of the potentiality for contraries

„This garment, for example, *may be cut in two* and yet will not be cut in two, but will wear out first. In the same way, it may not be cut, for it could not wear out first were it not possible for it not to be cut in two. This holds for all other events as well which are mentioned as having the same kind of potentiality.“

excursion: *privation*

For Aristotle, privation is if a thing is hindered in fullfilling its potential.

To what is this kind of potentiality a *proper* potential (to whom or what does it belong)? What could possible „fullfill“ it? Or is it not subject to privation at all?

Privation is what many (modern) theories hold as constitutive for „the human“, and for ethics. The idea of socalled *Mangelontologien* (ontologies of lack) is that only through affirming privation – the being hindered through communality in fullfilling exhaustively ones potential – can we live together socially.

my suggestion: abstraction as a method

(no Royal Road!)

this is *the kind of properties proper to abstract entities* like a community, a constitution, plan, a generalized concept like a form or a schema, etc. The more such potentiality an abstract entity (an artefact) has, the higher its value for societies.

Abstraction allows to conserve potentiality!

⇧

to have a *potentiality for contraries* is a *meta-property* for Aristotle.

Not MANGEL ONTOLOGIE: *abstraction creates an excess of potential properties for its subjects. It introduces an economy principle*

the degree of individuation (freedom) with which the mastering of abstractions can govern its subject matter.

being-in-act
being-in-potency

can algebraic invariance play the role of Aristotle‘ essential today?

essential

no contradictions allowed in the essential

composed and divided yes, but not distributed!

(the properties belong to a subject (thing))

ARTICULATION

accidential/probable

contradictions are operationalized within the dynamics of privation as contradictories the can be *conjuncted* and *disjuncted*

no!

(the properties belong to an event)

Contingency in the universe due to dynamics of privation.

Aristotle economized the principality of his necessitarianist predecessors!

for Aristotle, Truth had a Nature!

This is the Reality-Principle of Aristotle!

„lending articulate voice to that with inarticulate eloquence“

principle of correspondence for accidentials

And the *Nature* of Truth (within Realist Philosophy in Aristotle)

[For Aristotle:]

„Accidental beings are not necessary but indeterminate, and their causes are unordered and infinite.“

topology constructs spaces by establishing relations of correspondence!

The principle of correspondence (at work in language) transmits the properties of the things and their causes to the statements about them.

> *correspondence is not representation but articulation!*

how to *term*
without *coercion*?

the idea of a natural flow we can tune in …

being is not *absolute*

L. *absolutus*, pp. of *absolvere* "to set free, make separate", "without reference to anything else, not relatively"

for Aristotle **but *numerous***

L. *numerus* "a number, quantity," from PIE root **nem-* "to divide, distribute, allot" (related to Gk. *nemein* "to deal out;"

Aristotle‘s Reality is analytical, of differential make-up which allows for generation and decay, transformation, becoming

quantification
quantization because of the simultanous existence of contraries

being-in potency can never be fully actualized.

motion
infinity
void
fullness

Reality actualizes from „linking up“ being-in-act (essences) and being-in-potency (accidentials, caught up in the dynamics of contrarity which is driven by potentiality, privation).

Aristotle‘s Nature

that which *expresses* inarticulate Elegance

cannot be exhausted by language, the *poetic* principle of the world

the domestication of Nature by Language

Tarski‘s Nature

that which *expresses* the appearances

can be determined by formal language, the *semantic* principle of the world

the domestication of Language by Algebra

What happens to the *inarticulate elegance* of that which appears without being forced into expression?

How can Algebra *domesticate* Nature *(de re)* and not only *coerce it into form and consequence* through controlling Speech *(de dicto)*?

the beauty of an equation does not appear if we see a solution, but from the promise of integrating differences without conflict

if we can see inarticulate promise in it!