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three addendums

articulating quantities

when things depend on whatever can be the case

by Vera Bühlmann

„Man can think in the sense that he possesses the possibility to do so. This possibility alone, however, is no guarantee to us that we are capable of thinking.“

- Martin Heidegger

„Further still, beyond the world of representation, we suppose that a whole problem of Being is brought into play by these differences between the categories and the nomadic or fantastical notions, the problem of the manner in which being is distributed among beings: is it, in the last instance, by analogy or univocality?“

- Gilles Deleuze

The world is everything that is the case – I would like to take this famous line from Wittgenstein‘s Tractatus as a starting point. The world is not the totality of things, he sais, but that of facts – I would like to consider, inversely, the world as the totality for whatever *can be* the case. After this inversion we can – a small deviation not withstanding – keep with Wittgensteins language game and call the totality of whatever can be the case the totality of *arte*facts. Artefacts capture and embody acts of concentration, not things that have happened or are given. What distinguishes them as *arte*facts from facts is that they conserve an *act of concentration* by condensating this act into manifest form.

My core interest in the following concerns the possibility of a philosophical grammar, in which the act of conception is ampliative. A philosophical grammar in which conceiving is engendering-by-inference. Ampliative inferences are, according to Kant, inferences *capable of broadening a terms extension beyond the possibilities that were contained in the premises*. Such thinking-as-conceiving cannot be captured by the synthetic/analytic distinction, as Kant was well aware of. It is not deductive, it cannot easily be argued as inductive, and it seems to involve an aspect of *inception,* of beginnings.

The brief sketch I would like to layout in this paper to support such an interest in artefacts as condensations of intellectuality departs from exploring a peculiar proximity between Ludwig Wittgenstein, Martin Heidegger and Gilles Deleuze. A proximity which stems from their common interest in the Kantian insight that reason conditions experience – an insight which, in a somewhat un-Kantian way, is explored by all three of them in relation to *acts of learning* rather than *objects of knowing*.

In their own individual ways, Wittgenstein, Heidegger and Deleuze have evoked the ancient sense of *mathesis* as an art of such conditioned learning. They have embraced the challenge that for learning, the conditions can never be sufficient nor clear and distinct. In such a sense of mathesis, to which I will refer to as *mathetical*, learning is less concerned with representation or recognition than with an act of appropriation and inhabitation of new capacities and abilities. Learning in a mathetical sense involves a kind of *privation* which inverses the usual sense of the word: it involves a privation which engages in a logic of *giving*, not of *depriving*.

I would like to extend on this aspect that for learning the conditions can never be sufficient nor clear and distinct. I will suggest to view artefacts in a broad sense — be it software, architecture, film, music, a piece of technology, suggestions for policies, tools for financing, business plans, recipes or theory books – as the manifest form of acts of learning. I will regard artefacts as cases – and dedicatedly *not* as singularities – as cases which are conceived and engendered by ampliative reasoning.

Artefacts, in such a view, are condensations from the outer space of intellectuality. They are aliens-from-within, if you like, they are popularized, and in that sense de-capitalized acts of concentration. If it makes sense at all to say that they *are* we can follow the leap from *being* to *existence*, and claim that just like things *are* insofar as they *are there*, artefacts *are* insofar as they *are here*. If *being* corresponds to things-as-they-are-named, and *existence* to things-in-their-thingness, we can say that *insistence* corresponds to the phantasmatics of things-in-their-pre-specific-objectivity.

1 Within the outer space of intellectuality

Wittgenstein had started to sketch out a philosophical grammar for addressing things-as-facts. A philosophical grammar for addressing things-as-*arte*facts considers things in their pre-specificity. It assumes they can be named, in this pre-specificity which they manifest, not by *grammatical names* but by *mathetical* names, i.e. by *polynomials*.

The *literal meaning* of polynomials is *having many family names*. Polynomials name heterogenous things, hybrids that comprehend aspects of many generic lineages. Polynomials name things that have no natural belonging – if natural belonging means that the identity expressed belongs to exactly one genus or genre.

The *mathetical context* of polynomials we can characterize as follows. For all sciences working with methods of probabilistics, factoring polynomials is as *ordinary*, as *elementary* and *capability-dependent* a practice as composing more or less well-formed, more or less well-reasoned arguments in sentences is. Polynomials feature in systems of differential equations, and they are especially useful when describing processes which do not unfold steadily in space and time, but with fluctuations. Polynomials allow to map, probabilistically, fluctuations wherever signals can be made out that come in patterns of waves. Polynomials are the building blocks of all sciences dealing with electrical technology – just like grammar is the building block for dealing with sentential structures.

Assuming such polynomial predicability of artefacts allows to see them responding to all the grammatical categories we know: *mode*, the *expression of tense, mood, voice, aspect, person, number, gender and case*. Taking this grammatical perspective does not mean to involve polynomials now into the socalled „linguistic turn“; but it does mean that there is some kind of "languagability" involved in probabilistic reasoning. Taking a grammatical perspective grants that factoring polynomials is not a purely mechanical procedure, but one which can be done with more or less intelligence.

If we take the algebraic formula for *a circle* as an example, we can see that what this formula names is *never fully given*. Polynomial predication is not directly about the assignation of an object as a particular thing. The formula for a circle is the formula for *any* circle, and needs further determination that cannot be deduced from what the formula contains in order to denotate a *particular* circle. The Polynomial space of predication organizes a space where things intermingle in the pure generality which objects have that are regarded in their dissolution into pre-specificity. There are no particulars involved in this intermingling of pure generality organized by the polynomial space of predication. There is nothing to be counted yet.

Such a grammar is a formulaic grammar, and the pre-specific identities it names are – to a certain degree – *evoked*. They are literally *called out, summoned,* a bit like *daimonions,* manifest voices, out of the outer space of intellectuality we all engage with when we learn. This space comprehends virtually *anything* that can be thought rigorously.

The attractive promise is that such a grammar which regards artefacts as its polynomial articulations may provide us with the ability for structural thought in domains where reason is not only *insufficient* but also *abundant*. In other words: whenever we refer to the probable. The probable, I would like to suggest, is the outer space of intellectuality where all the artefacts that ever *were*, *are* or *will be here*, are conceived, breeded, and engendered. Algebraic structuralism, i.e. structuralism within *conditions of abundant and insufficient reason* concerns the genesis of acts of reasoning.[1]

What I would like to present in the following is a collection of loosely integrated lines of thought to approach such a way of thinking about the articulate-ability of quantities by polynomial predication.

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[1] If we conceive of it in terms of a grammar, we can avoid subscribing to what Hegel had called „the bad infinity" – which would mean ascribing artefacts the status of an absolute self- or auto-conditioning. Though infinitary in regard to its engagement with an element of the probable, such a grammatical structuralism is nevertheless finitary in the actualizations it conditions. We can express what the structure allows to be the case.

2 mathesis

There was a time when the *theory of the forms of experience* and that of *the work of art as experimentation* had maintained an intimate relation. In a somewhat outdated sense of the word, *the arts* were referring to the development of *abilities* very generally, to a sort of cunning reason and the sophistication in *how* we can carry out human endeavors in general. As such, the term comprehended a double make-up of the development of such abilities as *ars* and as *techné*. The Greek term *techné* seems to have been applied in a somewhat sophistical, pragmatical sense for comparing such developed or cultivated abilities. In its Latin translation as *ars*, this sophistical dimension was largely reoriented towards a more meditative frame of reference. In both cases, however, *techné* and *ars* were meant in a more general sense than any skill or craft in particular. And even more importantly, they both implied an infinite scale: there can be no comprehensive definition, no delineation of *how good we can learn to be in something*.

Abilities as abilities, both in *ars* and *techné*, cannot be mastered, strictly speaking. Developing them means learning, in a non-transitive sense. Today we have largely dismissed such a notion of learning, in favor of giving an objective dimension to knowledge which we can learn to cultivate by what is today called *literacy*. Yet different than the old notion of *mathesis*, *literacy* is consitutively entangled within a logic of recognition. Attending to the literal assumes there is a naturalness of meaning to it. This leaves us with an insuperable helplessness when dealing with the *fertility* and *autonomy* of thoughts thought , which they acquire within the outer space of intellectuality.

Heidegger has paid attention to this alternative to the notion literacy. He referred to it with cautious consideration as *the mathematical,* and meant by it, in an open sense, *that which can be learnt*. In *Die Frage nach dem Ding* (1935/36), Heidegger comprehends genuinely philosophical thought as thought revolving around the notion of *the thing*. The mathematical is concerned with things, he says, insofar as we can learn about things. Not simply how to use them, name them, or master them, but rather how we can learn about things in their thingness. About bodies as bodiliness, plants as plantness, etc. With these abstract terms Heidegger does not refer *to an idea* of a thing, but to a certain kind of intellectual experience of an object as a thing in a certain appearance. This experience is conditioned by a sort of intellectual intuition, yet it only concerns the possible awareness of our interiority. Such experience is not real, for Heidegger. Rather he introduces the mathematical as that which at one and the same time gives things to us, and allows us to learn about them: „*The mathematical, this is what we intrinsically already know about things, what we do not have to extract or abstract from things but what we, in a certain way, bring along ourselves*“.[2]

Learning, he continues, is *a giving to oneself what one already has*. It contains an element of *a-substantiality* which for Heidegger is, in this certain mathematical way, strictly personal.

3 There is a naturalness proper to reasoning

Different than Heidegger, Gilles Deleuze has suggested to consider a possible generalization of this peculiar *element of a-substantiality* that is involved in mathetical learning. He conceives of *purity not as an attribute, but as an elementarity, as a transcendental quasi-naturalness* proper to reasoning. This elementarity, for Deleuze, is conditioned by three inseparable principles: that of *pure quantitability*, complemented by those of *pure qualitability* and *pure potentiability*. Raising these terms, the quantitative, the qualitative and the potential to the level of *-abilities* is crucial for understanding Deleuze. He calls them principles, but manages to call them such – by raising them to the level of abilities – in a way that they don‘t *presuppose* anything *given*. These principles don‘t allow us to recognize, imagine or picture ideas by thought. Rather, ideas bath in this pureness, and this pureness grants that thought is *natural* in a different way from assuming its Good Nature. Within such a setting, ideas need to be indexed. They need to be actively coded. This is how thought can engender thinking, within this elementarity of pureness. Deleuze conceives of ideas as the differentials of thought, and thinking means determining – reciprocally – the differential relations contained within them. Thus, Deleuze presupposes *a*

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[2] „Das Mathematische, das ist jenes an den Dingen was wir eigentlich schon kennen, was wir demnach nicht erst aus den Dingen herholen, sondern in gewisser Weise selbst schon mitbringen“

naturalness for reasoning, which precedes the assignability of truth or falseness to any act of thought in particular. This naturalness itself provides neither sufficiency nor well-foundedness for emerging thoughts.

Considered together, Deleuze‘s ideas and this elementarity of pureness make up for considering reason not from the point of view of its conditioning, but from the point of view of inception or its genesis, as Deleuze called it. Precisely because he assumes a naturalness to reason, Deleuze can hold, in a *mathetical* sense, that reasoning depends upon learning.

Deleuze inverts the analytical assumption of an objectivity of problems. There is an objectivity of problems, for him, but it is given as Ideas – of which he conceives as Differentials. Hence, we cannot have *representations* of problems, we can only *formulate* them. Reasoning, for Deleuze, is the faculty capable of formulating problems in ways that allow for Critique, and this means: *formulating problems-in-general.* This way of formulating problems-in-general, I would strongly like to argue, is algebraic and symbolic – not literal or numeral in any direct sense.

But let us look more closely at this articulate-ability of quantities within such a transcendentally-empirical setup. A Differential takes the fractional form of a ratio. If ideas are not what is represented or mapped in reasoning, if they are differentials which need to be formulated – in order to pose the problem whose objectivity they embody – we cannot deal with a differential‘s fractional form as a ratio directly. We have to empirically-experimentally investigate the ratio (*think: the idea, the differential*) by expressing it in a variety of forms. This is what polynomials allow for. Polynomials are algebraic ways of how to *index* ratios such that they can be put into symbolical terms that allow for a variety of ways of how to express the ratio‘s quantity. As algebraic expressions, ratios are put into an arrangement of terms which involve *indeterminate variables* and *constant values.* The sum of these terms either needs to equal zero, or another version of the same quantity articulated differently, i.e. factored differently. Like this, ratios can be algebraically expressed such that they can be determined strictly reciprocally. The identity postulated by algebraic equations, is expressed as a relation of idem-potency, as George Boole had called it. Algebraic expressions with polynomials allow for all the arithmetic operations except division – and division is precisely what is expressed in the form of *ratios*. Like this, Deleuze can maintain that ideas can be tested – by algebraically articulating the form as a differential, i.e. as a form which comprehends a fully determinable a ratio.

He has extracted the philosophical consequences of this when he writes that *quantitas*, the Kantian concept of the understanding capable of grasping the *quantum*, i.e. things-in-their-extension, cannot be regarded as powerful enough for dealing with the different kinds of generality at stake: „*The zeros involved in dx and dy express the annihilation of the quantum and the quantitas, of the general as well as the particular, in favour of 'the universal and its appearance'.*"

The assumption of *abundant yet insufficient conditions for reasoning* allows for an empirical science of investigating the universal and its appearances. The particular and the general come to be, therein, the toolbox of such experimental measuring – concepts are representing the objectivity of something problematical only insofar as they are tools for learning to think. Deleuze degrades Kantian concepts of the understanding quite plainly in favor of such learning: „*As a concept of the understanding, quantitas has a general value; generality here referring to an infinity of possible particular values: as many as the variable can assume.*" So far so good, but he continues: "*However, there must always be a particular value charged with representing the others, and with standing for them: this is the case with the algebraic equation for the circle, x2 + hold for ydy + xdx =0, which signifies 'the universal of the circumference or of the corresponding function'.*" The algebraic formula for a circle needs a symbolic investment in order to become apparent as a particular circle. The particular, hence, is not a given concrete but an evoked appearance. An appearance engendered through a kind of abstraction which is purely symmetrical, it creates consistencies by testing the reciprocal determinations of differential relations. We have to dramatize ideas, as Deleuze calls it. The generalities are what can be extracted from abstract thought, not the other way around. Abstract thought does not presuppose the General Forms as given. Thus, the validity of General Forms can only be empirically grounded. Concepts can be created *mathetically,* they are grounded in what we have learnt to conceive rigorously.

Just like in the case of Heidegger, for whom such mathetical learning as *„giving to oneself what one already has"* is strictly personal, also Deleuze's notion of reasoning as learning is enacted by Personas. But for Deleuze, attending to the thingness of things means attending to ideas within the outer space of intellectuality – and this is only possible if we actively dramatize them. Both, Heidegger and Deleuze assume a dynamism which allows such attending or dramatization. For Heidegger, this dynamism takes the mechanical and in that sense self-sufficient form of a proof which pivots around the given axis of time. This self-sufficiency is opened up by Deleuze. He allows the mechanical, linearly circular dynamism – Heidegger calls his notion of proofing *Kreisgang* – to follow lines of flight which always depart from what has just been learnt.[3]

3 A *locus in quo* of imaginary points and figures

Let us raise some of the background issues to algebraic numbers and symmetrical quantities.

In 1883 Arthur Cayley, a British algebraist working on variational calculus and invariance-theory, gave his presidential address of the British Association for the Advancement of Science in London with the following endeavor. There is a notion, he told his fellow intellectuals, which is *„really the fundamental one (and I cannot too strongly emphasize the assertion) underlying and pervading the whole imaginary space in geometry."* It is hard to see at first what this statement implies, and why he holds it of such importance to devote his entire speech to it and this with such a tone of gravity in his voice. Has not geometry, at least since its analytic

[3] Deleuze opens up, with this characterization of elementary pureness of reason, in which thought dramatizes ideas through involving them into spatio-temporal dynamism by specifying those dynamisms, sights onto a philosophical domain that is conditioned by abundant and insufficient reason. I would like to suggest that this is the domain of symbolic algebra, and that the cases engendered by it – the artefacts – make up all the spatio-temporal dynamisms, in the plural. Every single artefact embodies a multitude of acts of concentration which were necessary for dramatizing an idea. It is in this sense that they conserve intellectual energy. They provide an energy which is neither finite and resource based, not infinite and unconditioned. Intellectual energy is encapsulated in artefacts of any kind, i.e. of any symbolic constitution, as the articulation of ideas by acts of concentration. The quantity conserved therein, however, depends upon our ability to articulate it. If we naturalize those artefacts, as autolog or self-sufficient objects without esteem for the acts of concentration they encapsulate, they don't conserve but consume energy. Because they establish comfort as a means to dispose of the necessity and benefit proper to acts of thinking and concentration, in favor for the establishment of a milieu of explicated, institutionalized Forms of General Intellect.

turn to the Cartesian Space of abstract representation, lost its cosmologically ordered elementarity in favor of merely providing an imaginary plane for experimental science? [4]

So what exactly is Cayley referring to with this imaginary space in geometry – what had happened?

The crucial sentence is the following specification Cayley gives: *„I use in each case the word imaginary as including real.*" Both terms, imaginary and real, are meant in their number theoretical sense, but nevertheless, the issue Cayley wants to address is not one dedicatedly for mathematicians. Quite to the contrary, his concern is: *„This has not been, so far as I am aware, a subject of philosophical discussion or enquiry"*.

The issue raised in this address concerns the grand question of whether and in what sense a notion of space is relying on experience and subjectivity. Yet the extraordinary take it presents, for philosophers, is that this question is raised out of the field of number theory. This is an unusual perspective. How can we, philosophically, conceive of space such that it features *„as a locus in quo of imaginary points and figures*", or in other words: *as the scene of the event of a peculiar kind of „elementarity" where figures are articulated out of a numerical domain* of which we must, somewhat paradoxically, trie to understand that it literally *„includes the real*".

By „including the real" is meant that the numerical domain at stake is said to extend beyond the infinite number line of the real numbers. In their continuity, the domain of the real numbers comprehends all the positive and negative integers, zero, as well as all the rationals and the irrationals. It is indeed difficult to *picture, mentally,* what could be left out by the real numbers, but this is precisely the point of Cayleys address.

[4] Cayley's is not a solitary voice at that time. Within the last decades of the 19th century, Gustav Lejeune Dirichlet, Ernst Kummer, Leopold Kronecker and Karl Weierstrass all wrote on the theory of numbers involving algebraic quantities; Husserl published his Habilitationsschrift entitled Über den Zahlbegriff; Dedekind wrote Was sind und was sollen Zahlen, and Algebraische Grössen, Russell wrote his PhD on the Foundations of Geometry and Whitehead published a comprehensive volume entitled Universal Algebra. All of this before the Principia Mathematica.

From the perspective of number theory, Cayley's question considers the possibility of a kind of intellectual intuition, and it considers that the quantitative may host something like *forms of construction* which might hive off such a notion of intuition out of the threatening swamps of unconditioned revelation in a mystical or theological sense.

The imaginary numerical domain Cayley is referring to is that of the Complex Numbers, and what this domain allows – as we could perhaps put it – is *operations on real infinities*. The crucial point about them is that their conditioning cannot be thought of as natural, if we understand by natural by its more conventional notion, not the Deleuzean one, namely that the quantities describing it need to be factorizable in a unique and necessary way, according to an assumedly universal order of primes.

This may seem like a fancy question for number-crunchers, not for political and intellectual realists, materialists, or idealists, but just consider that none of our electronically maintained infrastructures today would be working without those quantities. And yet, their usage is still today commonly put into rhetorical brackets which claim that only the „real" part of these operations was of importance, philosophically, whereas the imaginary part is called „but a technical trick" which we can apply when dealing with symbols. Contrary to this view, Cayley raised the question concerning the „nature" of such tricks.

Can there be, in short, something like an intuitive rendering-present by intellect, such that we can learn to say something reasonable about the conditions of this rendering-present – even though we cannot assume any necessity for it to appear as it appears?[5] What was preoccupying Cayley, and many others in the second half of the 19th century, was the unsettling suspicion that we cannot exhaustively address reality by investigations following the Cartesian formula *verum et factum convertuntur*. The status of numbers has grown problematical in a new way, with this newly developed capacity to render-present, symbolically and insofar intersubjectively, by acts of intellection.

[5] This is indeed a very old meaning of symbols – symbols evoke an immaterial presence in our thinking of something which lacks manifest presence. Symbols are place-holders, indexes, and they enforce a certain immediacy upon us. Hence our associations with symbols tend to center around mystical or sacral contexts. Or around contexts of undisputable control, when we think of our passports as symbols of our identity for example.

4 Considering the symbolicalness of symbols

The troubling question can be put like this: *how can we conceive of the symbolicalness of symbols in Universal Algebra?* For Whitehead it was an open question. For Russell just as for Husserl, it was clear that assuming for symbols a status of their own – one that is not grounded in geometry nor in arithmetics nor in language – would be profoundly misleaded; they both held firm – albeit in different versions – that symbols need to regard *necessary facts*.[6]

Yet with algebraic expressions, there is an objectivity proper to symbolic encodings that allows the encoded to be referred to and represented *in purely general terms*. This generality is not gained by strictly deductive reasoning, and it nevertheless does not depend upon psychological subjective experience.

Conceiving of a genuine symbolicalness of symbols means tackling with the primacy of abstract algebra as the means for formulating symbolic constitutions. These constitutions provide the structures for what can be expressed as the cases of this peculiar algebraic generality.[7] Strictly speaking, the fundamental theorem of algebra leaves the general applicability of arithmetics problematic. If algebra is granted a universal status, applying arithmetics turns into a practice of engendering solutions as cases, i.e. of calculating solutions which are not, strictly speaking, necessary solutions. [8]

For the majority of philosophers, an affirmation of this would be a straight forward capitulation of enlightenment philosophy at large, because it means that the strong link between calculability and necessity were broken, and along with

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[6] (Anschauungsthatsachen for Husserl, Logical quantification for Russell)

[7] We calculate spaces with algebraic numbers which relativize the assumed unproblematical rootedness of all numerical values in positive natural integers or a homogenous spatiality, and with regard to the formalization of language, we very well know meanwhile about all the problems related both, to universal and to existential quantification.

[8] Algebraic terms are polynomials, they embody unequal potentials. The liberty of engendering solutions as cases comes in because every algebraic solution requires a depotentialization of its terms, such that an order can be established which is shared by all the components. If number spaces that may extend into the imaginary and algebraic ideality are allowed for solving polynomial equations, there are many different ways of achieving this depotentialization. Hence the vastness of the possible solution spaces.

that, the distinction between *philosophy as metaphysics* and *philosophy capable of critique*.

Yet if Algebra‘s universal status is considered as complementing a probabilistic element, into which the formula – i.e. the algebraic *identity-as-relation-to-be-established* – is seeked to be integrated, all that the fundamental theorem of algebra asks for, philosophically, is to ascribe a different modality to the abstract objects of mathematics and logics than that of *necessity* or *contingency*.[9] I read Deleuze‘s concept of the virtual in these terms, as the modality for the experienciability of things which are not merely possible but real. *Virtually real* means in principle fully determinate yet never actually exhaustively determinable. We can consider the virtual as the modality of the things engendered by abstract thought. The symbolicness of symbols encodes forms of structure for determining unknown quantities, and is itself neither form nor content. Such algebraic quantity-expressions can be considered „pure“ in a quasi-Kantian sense: They make reference to no specific magnitudes at all and work only with conceptual definitions. *„It sets before the mind by an act of imagination a set of things with fully defined self-consistent types of relations*“ (p. vii), writes Whitehead about such vectors of imaginary verticality.

5 Coda

Aristotle had performed a bold move when he appropriated from the Olympian Gods their mark of distinction, their family names as a sign of belonging to different generations and genera, and claimed this divinely distinctive mark to be applicable to all there is. *All there is is things that can be named*, he set out to consider.

Once people had started to conceive of the mythical Happenings in terms of philosophical *consequences and inferences between things that can be named*, a

[9] Algebraic reasoning means specifying what outcome you will want to have, and producing it by taking an infinitary approach to deal with this probabilisitc element. Infinitary means, by eliminating from the vast and non-controllable possibility space of your solutions anything you do not want to feature in the result. This elimination procedure works by injecting into equations, as a kind of doping, whatever is necessary and sufficient for common denomination and factorization of the terms involved, such that the two sides of an equation may be transformed and balanced in ways that are neither fully necessary nor arbitrary.

structure was needed to receive and conserve the voices of the Mythical Personas. Words originally simply meant verbs, abstract acts in infinitive form. *Energeia* was Aristotles term such an abstract principle of Actuality. With the verbs, grammar was providing a structure to receive and conserve the mythical voices by distinguishing cases, as a sort of a negative form, in which we can encounter *things-that-can-be-named* as affected by *energeia*, as involved into the actuality at play with the verbs.

We still commonly say today – albeit we mean it, undoubtedly, in a largely technical and sterile sense – that grammatical cases are the structures provided to receive and express what is *decadent*, what is falling or declining. This is where the term *casus* comes from. Language and its grammar solves the threat of decadence for community by turning it into a problem to be articulated. As such, it needs not be *solved* anymore. The effect of expressing the threat in language literally *dissolves* it, by probabilizing the forms in which it might appear. These articulated expressions have led a fertile live, within the space of probability and intellectuality. Entire populations of words have been conceived, engendered, and raised, which allow for this enormeous richness in articulating *what may be the case*.

The real metaphysical question today is not about Being‘s analogy or univocity. It asks us to actively value and esteem artefacts as the conditions for everything that can be the case. The real metaphysical question today is how we can account for polynominality and the spatio-temporal dynamisms they engender.

postscripts

1 Algebraic conception, or the sexuality of the Cut

Let us extend more on the particular context in which Cayley‘s address can be located from today‘s retrospective. The full passage of what we have cited goes as follows:

"In arithmetic and algebra, or say in analysis, the numbers or magnitudes which we represent by symbols are in the first instance ordinary (that is, positive) numbers or magnitudes. We have also in analysis and in analytical geometry negative magnitudes; there has been in regard to these plenty of philosophical discussion, and I might refer to Kant's paper "Ueber die negative Grössen in die Weltweisheit (1763)", but the notion of a negative magnitude has become quite a familiar one, and has extended itself into common phraseology. I may remark that it is used in a very refined manner in bookkeeping by double entry.

But it is far otherwise with the notion which is really the fundamental one (and I cannot too strongly emphasize the assertion) underlying and pervading the whole imaginary space (or space as a locus in quo of imaginary points and figures) in geometry: I use in each case the word imaginary as including real. This has not been, so far as I am aware, a subject of philosophical discussion or enquiry. As regards the older metaphysical writers this would be quite accounted of by saying that they knew nothing, and were not bound to know anything, about it; but at present, and, considering the prominent position which the notion occupies – say even that the conclusion were that the notion belongs to mere technical mathematics, or has reference to nonentities in regard to which no science is possible, still it seems to me that (as a subject of philosophical discussion) the notion ought not to be thus ignored; it should at least be shown that there is a right to ignore it. " (p. 784)

Indeed, Cayley appellation to philosophers for attending to the imaginary units was not without impact. When asking ourselves what the relevance of all of this might be to us today, it is important to be aware that the number theoretic take on the problem of space and experience has transversed nearly all the different camps, from phenomenological schools to analytical ones, around the turn of the last century. It is often forgotten that Husserl, Whitehead and Russell all started out writing on this subject before the splitting into different *vectors of valuing thought* have emerged, i.e. before the publication of the *Principia* of the latter two, and before the phenomenological writings of the former. Let us give a brief diagrammatic sketch through the larger context around the turn of the last century.

Whitehead had published his *A Treatise on Universal Algebra* in 1898, in order to present „a comparative study of the various Systems of Symbolic Reasoning" that had been allied to ordinary Algebra since mid 19th century. Those Systems of Symbolic Reasoning, as Whitehead calls them, had been looked upon „with some suspicion" by mathematicians and logicians alike: „*Symbolic Logic has been disowned by many logicians on the plea that its interest is mathematical, and by many mathematicians on the plea that its interest is logical*" (p. vi). The *dazzling status of symbols* in these newly emerging branches of Algebra, between concept and number, logics and mathematics, is so important because while the former is judging *existential imports*, or „content-conceived-and-formally-expressed"– what we commonly distinguish today as predicate logic from propositional calculus – the latter, strictly mathematical way of symbolic reasoning is held to be concerned with conventional definitions only, without existential import. The *dazzling status of symbols* had been widely neglected because in the emerging fields of Symbolic Reasoning, to shortly summarize Whiteheads argument and motive for writing his Universal Algebra book, it cannot clearly be distinguished anymore whether a statement is to be treated as a mathematical statement or as a logical statement.

Also Russell has published, prior to his work in the *Principia,* on these confusions. He has given one of the most informative accounts of what was going on throughout the 19th century in terms of an Algebraization of Geometry, and the diverse „metageometries", as he calls them, that had emerged on this basis. In his dissertation *An Essay on the Foundations of Geometry* (1897) Russell is concerned primarily with reconsidering the notion of the Kantian a priori, and its distinction between „the necessary and the merely assertorical", in such a way that a notion of knowledge may be maintained that is absolutely free from any psychological or empirical uncertainty. On the other side, Husserl had completed his Habilitation with a study on the notion of Number in terms of Psychological Analysis (*Zum Begriff der Zahl, psychologische Analysen,* 1887 – ten years before Russell and 11 years before Whitehead), and much later in his career he returned to this issue and devoted a study on *The Origin of Geometry* (1936).

Even after the advances in propositional calculi, in formalizing propositions as function and classes as sets, after the introduction of the notion of the „incomplete symbol" by Russell and Whitehead as a kind of a bridging principle across levels of ramification and between types, even after Wittgensteins Tractarian semiotization of the incomplete symbol notion into a sign-symbol dynamics which gives primacy to on *the rôle of learning* instead of *representing a piece of knowledge,* even after Heidegger's *Sein und Zeit* 1927, the algebraic symbolization which had broken free within mathematics and logics could not in any way be regarded as settled.[10]

In *The Origin of Geometry* Heidegger's teacher Husserl asked about the premises for the commonly accepted practice according to which every geometrical figure can be defined algebraically, by an equation from which – so the problematic practice – *„it shall be possible to infer directly from algebraic relations to geometrical ones, without any chance for habits of thought to impose themselves with the necessity of intuitive facts*", as he put it. The algebraic-analytical methods calculate directly with what was then called „pure quantities", quantities which supposedly do not need the assumption of intuition for working out their proofs rigorously. Whitehead had also commented on this generalization of the quantity notion: *„The introduction of the complex quantity of ordinary algebra, an entity*

[10] The original titel hence was *The Origin of Geometry as intentional-historical Problem*, it appeared in the same year as Heidegger's reflections on the mathematical from which we started out. This late book of Husserl served as a starting point for Derrida's endeavor to formulate a fully generalized logic, a kind of a logified logic, one crafted not by mathematization and symbolization but by by welding together grammar and logic into a non-negotiable bunker, barring existential import, in short: quantification, by sentential judgement (not conceptual judgement!) into absentia from legitimate discourse altogether. For Derrida, this is the necessary condition for treasuring metaphysical indeterminacy against the threats of positivism. If I had time to expand on this, I would have liked to point out an almost tragic complicity between the two stances – grammatology and positivism. Yet with regard to my argument here, which aims to plot out the main pillars for considering the articulate-ability of quantities, such an excursion is of minor interest and would be nothing but commentary.

which is evidently based upon conventional definitions, gave rise to the wider mathematical science of today. The realization of wider conceptions has been retarded by the habit of mathematicians, eminently useful and indeed necessary for its own purposes, of extending all names to apply to new ideas as they arise. Thus the name of quantity was transferred from the quantity, strictly so called, to the generalized entity of ordinary algebra, created by conventional definition, which only includes quantity (in the strict sense) as a special case“ (p. vii/viii). Conventional definitions, for Whitehead, refer to mathematical definitions.

It is important to realize that there were, at the time, two competitive vectors emerging. One which claimed a possible *mathematization of all of logics*, on the assumed basis that *all inferential necessities ought to be dissolved into a probabilistic framework* where we can at best, as we know it from physics, look for regularities as natural laws by a combination of mathematical and empirical inquiry. Of such characteristics are the *Laws of Thought* which George Boole had in mind when he developed the kind of symbolic algebra in 1854, which, in combination with IT, brings us the powerful and pragmatic practical operability with formal languages as we know it today. In theoretical terms, however, the competing vector has been much more successful sofar, namely *the logification of mathematics* in the tradition of Gottlob Frege up to Carnap, but affecting also Heidegger and featuring Badiou perhaps as its latest keeper of that Grail.

The secret danger that is being kept or contained, in this grail, is the susceptibility of the argument which related *calculability – and especially, today, computation – with necessity* to the adventures of abstraction. This argument had always been a philosophical-political argument, and it is such also today. Only, it had seemed for a short time span as if it could be politically sentenced into a proper place, if only it were guarded safely enough within stacks and stacks of complicated and formalized generalizations from the pleasures and fertilities of abstract thought and learning.

Whitehead, in any case, was well aware of this susceptibility of the necessity argument when he explained the confusion around the status of abstract algebra‘s symbols, which works with mathematical definitions that take the form of propositions, i.e. they embody an act of inference, a subject-predicate relation –

with the crucial difference, however, that different from logical defintions, mathematical concepts are never concerned with a transitive, in the sense of extensive, dimension of their definitions. This is what makes up their abstract symbolicness: neither form nor content, in any direct sense, they are something like *the encoding of a form of structure for an unknown quantity*. As such, as an encoding of a form of structure – notice the indefinite articles in both cases, *an* encoding, and *a* form of structure – algebraic quantity-expressions are „pure" in a quasi-Kantian sense, they make reference to no specific magnitudes at all and work only with conceptual definitions. *„It sets before the mind by an act of imagination a set of things with fully defined self-consistent types of relations"* (p. vii), writes Whitehead and distinguishes such conventional mathematical propositions from logical ones by pointing out that the former make no existential imports what so ever. The troubling question hence, regarding the symbolicalness of those symbols is, that those propositions take the same form as logical propositions, and criteria for distinguishing them both is what is at stake. One, after all, is expected to regulate our statements about the world, while the other is not commonly ascribed a direct regulatory rôle in reasoning. Without clarifying this, any algebraic take on geometrical and also mechanical, technical questions would *rely on nothing but analogy,* on the assumed affinity of those abstractly created entities with the properties of real existing things.[11] In effect, this means that the link between calculability and necessity were broken.

For Husserl with his strong methodological sensitivity, hence, an interest in the status of those signs as a genuinely symbolic one was profoundly misleaded. For him it was clear that every logical practice centers around the distinction between necessity and contingency, all reasonable dealings in the form of conceptualization and propositions, in short: all *acts of inference* need to be grounded on some intuitive facts.[12] Algebraic-analytical methods, so Husserl held, are not actually working with concepts and propositional forms, they are merely making use of what he calls *Hilfsbegriffe, auxiliary concepts,* like that of the

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[11] with that the generality of their expressed identity (which would be: the determined unknown quantity) is genuinely heterogenous, as the name polynomials, many-nominal, directly points out. Unknown quantities, expressed as a determined identity in algebraic form, take their „origin" in a symmetry break. Of such a heterogenous character is the generalization of quantity which Symbolic Reasoning was practicing: it became according to the indexes which the specific code allows for.

[12] These must be accounted for by other means than those provided by the spatiotemporal a priori which Kant had established for legitimating the status of inferences as judgements .

imaginary, the irrational, the continuous, or the differential and the integral. All the mathematical concepts which involve them – hence any definition which makes reference to one of these auxiliary concepts – must be made dependent on elementary arithmetics, i.e. on a *logical* definition of numbers. „*All the complicated and artificial constructions, which are also called numbers, the fractions and the irrationals, the negative and the complex numbers, have their origin and permanence in the elementary concept of numbers and the relations defining those; along with the latter, also the former would fall, yes, mathematics at large would dissolve. Every philosophy of mathematics ought to start out with an analysis of the concept of number*" (p.8).[13]

The question, hence, can be put like this: how can we conceive of the symbolicness of symbols in Universal Algebra? For Whitehead it was an open question, for Husserl it was clear that they need to regard necessary intuitive facts (Anschauungsthatsachen). For both of them, although in different ways, the ghost of Aristotelian symbols-as-marks-of-affections-of-the-soul seems to have reappeared. With the troubling difference that symbols have, in difference to Aristotle, turned into letters that can encode arbitrary content. As such they cannot only present a gone by experience anew, mentally, but they can provide a general structure for such „presentability" – which thereby becomes separated from the former basis on individual, lived experience. With algebraic expression, there is an objectivity to symbolic encodings that allows the encoded to be referred to and represented *in purely general terms*, independent of any individual experience. Such is, since the Cartesian paradigm, the character of objective reasoning. Symbolic algebra treats unknown and known quantities in order to determine the *unknown* in such a way that a problem – once it is put into an algebraic expression – can be solved.

In the 16th century, François Viète had stated what algebra, in his eyes, is all about in the following way:

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[13] „Alle die complicirteren und künstlicheren Bildungen, die man gleichfalls Zahlen nennt, die gebrochenen und irrationalen, die negativen und complexen Zahlen, haben ihren Ursprung und Anhalt in den elementaren Zahlbegriffen und den sie verbindenden Relationen; mit den letzteren Begriffen fielen auch die ersteren, ja fiele die gesammte Mathematik fort. Mit der Analyse des Zahlbegriffes muss daher jede Philosophie der Mathematik beginnen." (p.8)

„ Analytical doctrine or algebra - called „cosa" in Italy - is the art of finding the unknown magnitude, by taking it as if it were known, and finding the equality between this and the other given magnitudes."

And two centuries later, Leonhard Euler: *„The purpose of algebra, like that of all parts of mathematics, is to determine the values of unknown quantities, and this is to be achieved by rigorous consideration of the conditions which are prescribed, and which are expressed in the form of unknown quantities. Hence, algebra is characterized such that it allows for demonstration of how we can find unknown quantities out of known quantities"*. [14]

The challenge Algebra poses to thought, hence, consists in inventing the conditions and assumptions necessary to establish the provability of what we wish to analyze. Let us at least touch on some of the issues at stake. A quantity was always thought to have a double make up, ever since Pythagoras, Platon, Aristoteles, Euclid and Eudoxos started to explicitly reflect on quantities: magnitude is called what can be measured, and it typically comes in different kinds which cannot easily – that is, *only by analogy* – be put in relation with each other; multitude, on the other hand, is the property of being enumerable, countable, and it is completely independent of any question of kind or nature. While quantities as magnitudes have proportion, quantities as multitudes have ratios. Or to put this differently, while one aspect of quantities ask how much, and hence looks at specific grounding and integration, the other aspect of any quantity asks how many, and looks at generalization.

As long as people did not calculate within the rational number space, the ratios of the multitudes have unquestionably been thought to label the proportions of the magnitudes. The order behind the proportionality was thought to be cosmological and/or divine. With the rise of higher algebra throughout the 16th and especially the 17th century, this very relation between magnitudes and multitudes, when treating quantities, becomes obscured: symbolic operations purely within the ratio's of the multitude helps to turn science as cosmology into natural science. It

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[14] "Der Zweck der Algebra, so wie aller Thiele der Mathematik ist, den Wert unbekannter Grössen zu bestimmen, und dies muss durch genaue Erwägung der Bedingungen, die dabei vorgeschrieben sind, und die durch bekannte Grössen ausgedrückt werden, geschehen. Daher wird die Algebra auch so beschrieben, dass man darin zeige, wie aus bekannten Grössen ubekannte zu finden sind." Euler, Grüßen, Teil II, S. 3, zieht in 4000 Jahre Algebra: Geschichte, Kulturen, Menschen von Heinz-Wilhelm Alten, S. 290.

would probably not be too dramatic a thing to say that what characterizes modern science most, is the release of a whole bunch of properties and characteristics on the level of magnitudes that were literally invisible and ungraspable before they could thus be calculated. Among the most important achievements brought about by the *modern rationalization of the proportional order of things* belong, in the political realm, advances in map-making, cartography, and the introduction of measuring standards more generally, in the economic realm the calculation with interests and capital, in the technical realm thermodynamics and the cleaning of fossil energy resources. In short, domestication of the rational number space meant that the order of things, on the level of magnitudes and proportionality, could hence be inferred from empirical experiments, and needed no longer be deduced categorically, dogmatically.

Science, thereby, has become political. It was now put in the service of the emerging new order of the nation states – a statement which can perhaps best be illustrated by the proclamation of the French Academy in 1788 that the classical problems of mathematics should no longer be credited within institutional science: „*The Academy took this year the decision to never again consider a solution for the problems of doubling the cube, trisection of an angle, squaring the circle and of a machine of claimed perpetual movement. Such sort of researches has the downfall that it is costly, it ruins more than one family, and often, specialists in mechanics, who could have rendered great services, and used for this purpose their fortune, their time and their genius.*“

By the time of the 19th century, the issues previously contained within metaphysical questions in mathematics have started to surface in a different guise: the question now came to be on which ground to distinguish reasonable research from non-reasonable research. Shall we seek for abstract, proof-theoretic foundations, or restrict ourselves to mechanically testable foundations? If the former, shall we assume for abstract reasoning a synthetic, explanatory role, as Kant had suggested, and if yes, on what grounds? Kant‘s relying on intuition had clearly become problematical by the new abstract objects involved by now, and by the reality of the technological devices they made possible. Or shall abstract reasoning be allowed an ampliative role? If yes, how can we call it analytic, then, in any non-creative sense, how can we still hold on to thinking analytical thought leads us to what we have to assume as necessary rather than contingent? Or shall we, after all, decide upon an intuitionistic, constructivist point of view which grounds analytical inventions, if they are to be credited acceptance out of purely pragmatical considerations, on the grounds of objectively reproducible mechanical algorithms and computation?

Symbolic algebra, let us summarize, treats unknown and known quantities in order to determine the unknown in such a way that a problem can be solved. Let me formulate the dilemma in the following way: If we grant symbolic quantities something like Kantian *purity*, we must account for their quasi-transcendental status. If we don‘t, we must account for the *purity of facts of intuition*. Let us now return to Heideggers‘ conception of the thingness of things as the genuinely mathematical, in which he saw the form philosophical thought needs to take in order to be philosophical. Heidegger‘s core interest with this characterization is to re-activate *the principally in-definit dimension of learning via the mathematical*, as opposed to that of ontological, representational knowing, which puts the mathematical merely in its subordinate service as mimetical recognition: „*The mathematical is that which is evident in things, the evidence within which we move, according to which we experience things, and according to which we experience things as such and such. The mathematical is our principal disposition with regard to things, within which we take things to be in the way they are given to us, must or ought be given to us. The mathematical is hence the primary premise for all knowledge of things*“.[15] Thus we might formulate our question regarding the articulateability of quantities with Heidegger: *Are those symbols to be taken with reference to their formalness, their numericalness or their literalness*?

2 From self-reference to articulations of idempotency: algebra and the relevance of a notion of the virtual

What I would like to suggest is to go with Heidegger beyond Heidegger and consider, before the background I have introduced, the symbolic make-up of

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[15] das mathematische ist jenes offenbare an den dingen, darin wir uns immer schon bewegen, demgemäss wir sie überhaupt als dinge und als solche dinge erfahren. das mathematische ist jene Grundstellung zu den dingen, in der wir die dinge uns vornehmen auf das hin, als was sie uns schon gegeben sind, gegeben sein müssen und sollen. das mathematische ist deshalb die Grundvoraussetzung des Wissens von den dingen.

thingness itself. How can we learn to conceive of a genuine *symbolicalness of those symbols,* that would not try to integrate them within either one of these? The 3 stances which refer to either formalness, numericalness or literalness suggest to give primacy either to geometry/mechanics, arithmetics, or a kind of logical grammar/grammatical logic. Conceiving of a genuine symbolicness of symbols, on the other hand, would give primacy to algebra. All the references evoked and quoted sofar are testimonials of algebra's superiority in abstractness over any of them: we calculate spaces with algebraic numbers which relativize the assumed unproblematical rootedness of all numerical values in positive natural integers or a homogenous spatiality, and with regard to the formalization of language, we very well know meanwhile about all the problems related both, to universal and to existential quantification.

So why then this resistance against algebra? Let me cite a quotation from Husserl which expresses the involved unease quite dramatically: „*All the complicated and artificial creations, which are also called numbers, the fractions and irrationals, the negative and the complex numbers, they have their origin and rootedness in the elementary concept of numbers and relations; with this elementary concept, also the former would fall away, yes, all of mathematics would fall away*".[16]
Strictly speaking, the fundamental theorem of algebra leaves the general applicability of arithmetics problematic. For the majority of philosophers, an affirmation of this would be a straight forward capitulation of enlightenment philosophy at large. Yet from a more relaxed perspective, all that the fundamental theorem of algebra asks for, is to ascribe a different modality to the abstract objects of mathematics and logics than that of *necessity* nor that of *contingency*.

I would like to opt for the latter possibility and suggest that peculiar modality proper to algebraic reasoning can be understood as virtuality. In this, I follow the position of Gilles Deleuze, who views ideas as the objective being of *problems-as-problems*. This stance gives up the metaphysical preoccupation with *being-qua-being,* or its modern logicist guise as *set-qua-set,* for a preoccupation with what we might call the problematicity of categoricity. What this means, in somewhat easier

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[16] Alle die complicirteren und künstlicheren Bildungen, die man gleichfalls Zahlen nennt, die gebrochenen und irrationalen, die negativen und complexen Zahlen, haben ihren Ursprung und Anhalt in den elementaren Zahlbegriffen und den sie verbindenden Relationen; mit den letzteren Begriffen fielen auch die ersteren, ja fiele die gesammte Mathematik fort.

terms, is the postulate of an ideality as the empirical grounds for an experimental science on what we can actualize out of the abstract. An experimental science within the conceptual, which aims at finding sustainable, integrateable and situation-specifically optimal representations of problems. Because problems can only be solved through the possibilities their representations allow for. If we chose a simple framing for a problem, we get a range of simple solutions. The more complex a framing we affirm, the more varied the solution space we find ourselves confronted with. Algebraic reasoning means specifying what outcome you will want to have, and producing it by taking *an infinitary approach*. This means, by eliminating from the vast and non-controllable possibility space of your solutions anything you do not want to feature in the result. This elimination procedure works by projecting into equations what is necessary for common denomination and factorization of the terms, such that the two sides of an equation may be transformed and balanced in ways that are neither fully necessary nor arbitrary. George Boole has invented a very general version of such an algebraic method – identity, for him, is neither sameness nor difference, it is a relation of *idempotency* and this relation is itself treated, algebraically, as the „unknown quantity“ to be determined. By the same kind of infinitary thinking, Richard Dedekind has provided the general procedure of what he called *The Cut* for delimiting and defining number classes. Deleuze has extracted the philosophical consequences of this, when he writes that *quantitas*, the Kantian concept of the understanding capable of grasping things-in-their-extension, the *quantum*, cannot be regarded as powerful enough: *„As a concept of the understanding, quantitas has a general value; generality here referring to an infinity of possible particular values: as many as the variable can assume. However, there must always be a particular value charged with representing the others, and with standing for them: this is the case with the algebraic equation for the circle, x2 + hold for ydy + xdx =0, which signifies 'the universal of the circumference or of the corresponding function'. The zeros involved in dx and dy express the annihilation of the quantum and the quantitas, of the general as well as the particular, in favour of 'the universal and its appearance'.“*

Proof theoretical approaches that follow an infinitary thinking, one that aligns itself with learning and not with knowing, in the tradition of Boole and Dedekind, and up to a certain extent also Heidegger, is capable of conceiving the universal and its appearance by means of creating „phantastical notions“, notions which are not gained by abstraction as extracting commonality from sensible particulars, but by abstraction as engendering: „*the limit no longer presupposes the ideas of a continuous variable and infinite approximation. On the contrary, the notion of limit grounds a new, static and purely ideal definition of continuity, while its own definition implies no more than number, or rather, the universal in number.*" Dedekind's procedure assumes continuity as ideality, yet a kind of ideality where we find general templates for application, but ideality as the principle for a reason which is not supposed to grant clarity and distinctivness, nor sufficiency, but *engendering*. „*Modern mathematics then specifies the nature of this universal of number as consisting in the 'cut' (in the sense of Dedekind): in this sense, it is the cut which constitutes the next genus of number, the ideal cause of continuity or the pure element of quantitability.*"[17]

The interesting twist the Deleuzean proposition makes to our dilemma regarding either the purity of symbolic quantities or that of intuition, is that it heaves us onto the next level of abstraction in that it conceives of purity itself as a quasi-empirical element, not as an attribute. Deleuze's pure *element of quantitability* is quasi-empirical because the way we can make encounters, there, is not by sensible experience but by intellect. Deleuze complements the philosophical notion of categories – literally from greek *kategorein*, ways of *speaking about* or *addressing things*, as a form of distribution of abstract universality – with a notion of concrete universals, which he calls phantastical notions. „*It is pointless to claim that a list of categories can be open in principle: it can be in fact, but not in principle. For categories belong to the world of representation, where they constitute forms of distribution according to which Being is repartitioned among beings following the rules of sedentary proportionality.*" The kind of partitioning possible by symbolic algebra cannot be conceived in such a sedentary way. „*That is why philosophy has often been tempted to oppose notions of a quite different kind to categories, notions which are really open and which betray an empirical and pluralist sense of Ideas: 'existential' as against essential, percepts as against concepts, or indeed the list of empirico-ideal notions that we find in Whitehead, which makes Process and Reality one of the greatest books of modern philosophy.*" From his element of pure quantitability, the continuous as ideal cause which can engender cases by an open kind of distributions which are articulated out of the

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[17] all of these Deleuze citations: Difference and Repetition, p. 172

abstract ideality, Deleuze conceives what he calls a nomadic proportionality for algebraic partitioning: „*Such notions*“, he says, „*must be called 'phantastical' in so far as they apply to phantasms and simulacra*“. These phantastical notions are distinguished from the categories of representation in several respects, he continues. First, he says, they are conditions of real experience, and not only of possible experience. Given their pure quantitability-as-ideality, the conditioning they engender, by the procedure of Dedekindian Cutting, does not extend beyond that which it conditions, as is the case for a possibility – which may or may not come into effect in reality. „*In this sense, because they are no larger than the conditioned, they reunite the two parts of Aesthetics so unfortunately dissociated: the theory of the forms of experience and that of the work of art as experimentation.*“

3 Conception as mathesis. A closer look at the encounter between Heidegger and Deleuze.

Let us read this statement[18] in direct relation to Heidegger. For he too, with the attention he pays to the thingness of things, has criticized philosophy‘s claiming of a determinable categoricity of statements. For Heidegger, as for Kant, what can be denoted by categories – with the exception of modality as a category – are objectively *given* and cannot open our understanding towards the thingness in which every object, in its givenness to us as a thing, is constituted. Of all the categories it is only modality, as his *Being and Time* will argue, which addresses *the thingness of things as substance*, because it refers to temporal determination. For Heidegger, the object itself (not as a thing) is a mere insistence, *a standing against* (Gegenständlichkeit), and the convertibility proper to an object-as-a-thing comprehends its determinabilities, the way or manner in which an object-as-insistence actually exists.[19] He gives an example: „*A philosopher has been asked: what is the weight of smoke? He replies: subtract of the weight of the wood burned the weight of the remaining ashes, and you get the weight of its*

[18] „In this sense, because they are no larger than the conditioned, they reunite the two parts of Aesthetics so unfortunately dissociated: the theory of the forms of experience and that of the work of art as experimentation.“ (Deleuze ...)

[19] Heidegger, Die Frage nach dem Ding, ca p. 235

smoke".[20]Conceiving of substance as an object's insistence presumes, he argues thereby, that even with fire, materiality-as-substance cannot decay. It is only affected, by fire, in its form. Heidegger takes this example to support his claim for his form of mathematicalness our determinations of substances need to take: „*It is not enough to back these determinations up by grounding them in a principle of insistence only accessible for an intuitive and shared feeling. This grounding needs to be proofed: 1) that and why there is something insisting in all appearances, and 2) that the convertibility be nothing else than a pure determination of the insistent, i.e. something which maintains to this insistence the temporal relation of a drawing-out, a pulling-out or extraction. At stake are the rules for determining*-in-act, yet only ever in an incremental manner. For him, no Being-there is conceivable which is not constituted by language. Language is the relationality of any relation, it is that which can show us ways into the thingness of things if we subject ourselves to trace its movements by performing the Kreisgänge of mathematical proof.

Whereas Aristotle had started out his writings on categorical predication by putting all the happenings on stage as things which can be named, Heidegger attends to things not in the way they can be named, but in the way they can appear to us in their thingness. It is impossible to ever achieve full predication of a thing attended to in thought. So for Heidegger, the stance of a philosopher is that of taking care of Being by attending to its Dasein. For him – and this is where we come back to Deleuze and his phantastical notions – we would be misguided if we were to assume something like concrete universals. The thingness of things is only predicate-able by a daring kind of *casting* which he calls *design* (Entwurf).

„*Where the casting of the mathematical design is ventured, the pitcher of this cast places herself on a ground which comes to be itself projected itself only in the design ventured. There is not only a liberation proper to mathematical design, but also a new kind of experience and designing of freedom itself, i.e. of the resumed*

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[20] „Ein Philosoph wurde gefragt: wieviel wiegt der Rauch ? Er antwortet: ziehe von dem Gewichte des verbrannten Holzes das Gewicht der übrgibleibenden Asche ab, so hast du das Gewicht des Rauches." p. 235

attachment. Within a mathematical design, an attachment is taking place to the conditions claimed.“[21]

Conceiving of these conditions, claimed by mathematical design, as concrete universals which allow for a different kind of distribution than the categories do, is the great achievement where Deleuze goes beyond Heidegger. Compared to Deleuze‘s phantastical notions, there is something fatalistic about Heidegger‘s predicatability of the thingness of things. Its pivotal element is time, and he conceives of its unfolding in mechanistic terms (time as the pivot of every proof) – the only philosophical predicate-ability of Being as Dasein is subject to the larger power of a kind of probabilistic historicity, which alone is granted the status of being real. Neither contingent nor necessary, Heidegger totalizes the possible as the only mode in which we can attend to Dasein. Philosophers therefore need to become the caretakers of being-as-being, its shephards.

Let me approach this point where Deleuze diverges from Heidegger‘s notion of mathesis as learning by turning to a distinction introduced by Jules Vuillemin in his book *Necessity or Contingency. The Master Argument*. He concludes his book with pointing out a recent distinction in the history of modal notions, between a probability and a probability-amplitude. „*Classical physics was content with the opposition ‚this particle passes through A‘ versus ‚This particle has the probability of π (the circular constant, the Kreiszahl) of passing through A‘. This opposition has nothing to do with ontology: it incorporates what is due to our ignorance into the determination of natural phenomena. Instead of attributing a property or magnitude to a physical system, we attribute it a disposition or propensity to have that property or magnitude. Probability measures that disposition or propensity that belongs to a a system-in-act.“* This is just akin to the situation laid out by Heidegger – his mathematical learning about Dasein has no ontological and metaphysical consequences. Conceptual determination of Dasein involves *that, what* and *under what conditions, generally,* something *may appear*, but the appearances of the conceived possibilites can never be effected or realized in an a

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[21] Heidegger, das Ding: „Wo der Wurf des matehmatischen Entwurfs gewagt wird, stellt sich der Werfer dieses Wurfes auf einen Boden, der allererst im Entwurf erworfen wird. Im mathematischen Entwurf liegt nicht nur eine Befreiung, sonder zugleich eine neue Erfahrung und Gestaltung der Freiheit selbst, d.h. der selbstübernommenen Bindung. Im mathematischen Entwurf vollzieht sich die Bindung an die in ihm selbst geforderten Grundsätze. Befreiung zu einer neuen Freiheit.“

priori manner, he says. These appearances can only be encountered, if it happens to appear. This encountering depends upon one‘s relation to the givenness of things one has already learnt; the rules are always rules of analogical correspondance for Heidegger.

„*These rules for searching and finding within appearances‘ relationality in Dasein – the Dasein of the not given one in relation to the given Dasein of the others – these rules for determining relations within the Dasein of the insistence of objects are the analogies of experience*“.[22] An analogy is a correspondence, it means for Heidegger the relation of the „why like this“ (wie-so). What is standing in corresponance, thereby, are again relations: *„analogy is, according to its original notion, a relation of relations*“.[23]

There are two kinds of analogies for Heidegger, mathematical ones and metaphyiscal ones. The mathematical ones can be constructed in their akinness – just like *a* is proportional to *b*, so *c* is proportional to *d*. If *a* and *b* are given to us in their proportionality, and *c* is also given, then we can determine d by analogy. The unknown quantity can hence be positioned by means of such mathematical design or construction. This is categorically different in the case of metaphysical analogies for Heidegger. Here, he says the relations are not purely quantitative, but also qualitative. Metaphysical analogies involve relations among that which is not akin to each other. By introducing this distinction Heidegger completely forgets that proportionality always involves magnitudes (i.e. specific, and in that sense qualitative quantities) as well as multitudes (i.e. the numerical ratios between magnitudes of the same kind) – he forgets or ignores completely the whole issue about which people like de Morgan, Boole, Whitehead, Russell and even Husserl had been so concerned about.[24] This is crucial for the argument

[22] „Diese Regeln des Suchens und Findens des Daseinszusammenhanges der Erscheinung – des Daseins der nichtgegebenen Einen im Verhältnis zum gegebenen Dasein der Anderen. Diese Regel der Verhältnisbestimmung des Daseins der Gegenstände sind die Analogien der Erfahrung.“ ca 231

[23] „Analogie heisst Entsprechung, mein ein Verhältnis, nämlich das Verhältnis des „wie-so“. Was dabei in diesem Verhältnis steht sind wiederum Verhältnisse. Die Analogie ist, nach ihrem ursprünglichen Begriff gefasst, ein Verhältnis von Verhältnissen.“ ca 231

[24] „Jenachdem was in diesem Verhältnis steht unterschieden man mathematische oder metaphysische Analogien. Im Verhältnis des „wie-so“ stehen für die Mathematik Verhältnisse, die - kurz gesagt - als gleichartige konstruierbar sind: wie a zu b, so c zu d. Wenn a und b in ihrem Verhältnis gegeben sind und ebenso c, dann kann nach der Analogie d bestimmt, konstruiert, durch solche Konstruktionen selbst beigestellt werden. Bei der metaphysischen Analogie dagegen handelt es sich nicht um rein quantitative Verhältnisse, sondern um qualitative, um solche zwischen Ungleichartigem.“ ca 231

Heidegger continues to build with this distinction between mathematical and metaphysical learning: *„Here [in the latter, VB] encountering the real, its presence, does not depend on us, but the other way around. We depend on the real. In that case, if the encounter of a Third happens, and the proportionality between the two in whose region this encounter takes place is given, the Fourth can nevertheless not be inferred as if it would present itself by this act of inference. According to the rule of analogical corresponance we can only infer the general relation between the Third and the Fourth.We are only indicated the relation between a given and something which is not given by the analogy, i.e. the instruction of how we can look for the not-given by departing from the given, and of the manner in which we ought to encounter it, if it were to appear.*"[25] The probability element involved, in Heidegger's reasoning, is characterized just like Vuillemin's distinction of a probability as opposed to a probability-amplitude points out: the probability is that of an attributed disposition or propensity, but one that expresses an ignorance in awareness (or givenness).

Let us now see what distinguishes a probability-amplitude, which is „something altogether different" as Vuillemin points out, and how it answers to the the irritations of the algebraist-philosophers. *„We can compare it to an embryonic probability"* he begins, *„as the inventors of the infinitesimal calculus compared the „moment" of motion to an embryonic motion that an integration would bring to a state of „whole" motion."* But the comparison limps, he continues. *„For the probability amplitude, which is generally a complex quantity, does not figure among the elements of reality."* This is the crucial point: probability amplitudes can only be expressed if we allow for the kind of imaginative expansion the algebraists were so excited about. Vuillemin continues more precisely: *„To obtain a probability we must multiply two conjugated probability amplitudes. This means that, when we attribute that amplitude to a system, it is attributed neither as an actual property or magnitude, nor as an actual disposition or propensity to having such property or magnitude, but as a purely virtual disposition or propensity to*

[25] „Hier hängt das Begegnen des Realen, seine Anwesenheit, nicht von uns ab, sondern wir von ihm. Wenn im Bereich dessen, was begegnet, ein Verhältnis Begegnender gegeben ist und ein zu einem der beiden Gegebenen Entschprechendes, so kann jetzt nicht das Vierte selbst erschlossen werden, derart, dass es durch solchen Schluss auch schon anwesend wäre. Vielmehr kann nach der Regel der Entsprechung nur auf das Verhältnis des Dritten zum Vierten geschlossen werden. Wir gewinnen aus der Analogie nur die Anweisung auf ein Verhältnis eines Gegebenen zu einem Nichtgegebenen, d.i. die Anweisung, wie wir vom Gegebenen aus das Nichtgegebene zu suchen haben und als was wir es antreffen müssen, wenn es sich zeigt."

having it.“ Vuillemin conceives of such probability by amplitude as „second order potentiality“, and as such it not longer refers to „an ignorance what might have some chance of being only provisional.“ Just like Deleuze, who holds that his concrete universals, distributed by the forms of his phantastical notions, not merely condition possible experience – in sofar his approach would be largely reconcilable with the Heideggerian *mathesis argument* – but real experience, Vuillemin holds that the second order potentiality is physical, and he describes the modality of it as virtual: *„It is physical. It describes nature.“*[26]

In developing his notion of virtual structuralism as a Philosophy of Reason which does not, like the Kantian and the Heideggerian, remain within the point of view of conditioning, but expands into the pure element of quantitability and towards the point of view of reason‘s genesis, Deleuze explicity refers to Vuillemins work, especially to *Philosophie de l‘Algèbre*.

With Vuillemin‘s work on mathematics, he writes, *„'structuralism' seems to us the only means by which a genetic method can achieve its ambitions. It is sufficient to understand that the genesis takes place in time not between one actual term, however small, and another actual term, but between the virtual and its actualisation – in other words, it goes from the structure to its incarnation, from the conditions of a problem to the cases of solution, from the differential elements and their ideal connections to actual terms and diverse real relations which constitute at each moment the actuality of time.“* This genesis evolves in the element of a supra-historicity, as Deleuze calls it. While for Heidegger, mathesis is an art of concepts which in order to get in touch with the real depends a) an our ability to recognize it if it happens, and b) on the revelatory event of the actual appearance of the conceived thingness of things. The mathesis of concepts Deleuze introduces is not an art in that sense. The interesting aspect about these concepts is not that they can express singularities: every poetical attending to a situation can, in principle, do that as well. It would be trivial – for both sides – to conflate philosophical concepts with poetic expression. Such a mathesis of concepts is capable of engendering real cases, i.e. it provides the real dispositions for things to express their thingness in ways that can be generalized. This means that such

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[26] Vuillemin, Necessity and contingency, p. 264/65.

concepts are capable of engendering cases, cases which are, algebraically speaking, *possible*.

The appearance of things fully depends on what can be the case. Such conception is perhaps better named as *mathetical* instead of Heidegger‘s *mathematical*, because of its algebraic symbolicity. It proceeds from the articulateable conditions of a problem to the cases of solution. For such cases, the distinction between conceptuality and materiality collapses and is no longer useful. In both forms they are articulations out of the abstractness of symbolical reasoning-conditioning. This is what makes the approach so valuable for thinking about computation, information, and logistical infrastructures.